

Soliton Solution for the Spin Current in Ferromagnetic Nanowire

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We investigate the interaction of a periodic solution and one-soliton solution for the spin-polarized current in a uniaxial ferromagnetic nanowire. The amplitude and wave number of the periodic solution for the spin current have the different contribution to the width, velocity, and the amplitude of soliton solution, respectively. Moreover, we found that the soliton can be trapped only in space with a proper condition. At last we analyze the modulation instability and discuss dark solitary wave propagation for the spin current on the background of a periodic solution. In some special cases the solution can be expressed by the linear combination of periodic solution and soliton solution.

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I. INTRODUCTION

The study of magnetoelectronics has received considerable interest for its technological potential application. Both theoretical and experimental investigations mainly concentrated on giant magnetoresistance are of fundamental importance in the understanding of magnetism and applied interest in the fabrication of magnetic devices. In metallic ferromagnets, the differences between electronic bands and scattering cross-sections of impurities for majority and minority spins at the Fermi energy cause spin-dependent mobilities [1]. The difference between spin-up and spin-down electric currents is called a spin-current. It is a tensor, with a direction of flow and a spin-polarization parallel to the equilibrium magnetization vector $\mathbf{M}(\mathbf{r}, t)$ as [2]

$$\mathcal{J} = \frac{P\mu_B}{eM_s} \mathbf{j}_e \otimes \mathbf{M}(\mathbf{r}, t), \quad (1)$$

where P is the spin polarization of the current, μ_B is the Bohr magneton, and e is the magnitude of electron charge. The vector \mathbf{j}_e tracks the direction of the charge current, $\mathbf{M}(\mathbf{r}, t)$ describes the direction of the spin polarization of the current, and M_s is the saturation magnetization.

When the magnetization directions in the systems are not collinear, the polarization directions of the non-equilibrium accumulations and currents are not parallel or antiparallel with the magnetizations. This gives rise to interesting physics like the spin transfer effect in spin valves. The dynamics of magnetizations [3] is then governed by the parametric torques due to spin-polarized currents and magnetic fields. This spin-transfer effect was theoretically proposed [4, 5] and subsequently verified in experiment [6]. Since this novel spin torque is proposed, many interesting phenomena have been studied, such as spin wave excitation [7, 8], magnetization switching and reversal [9, 10, 11, 12, 13, 14], domain-wall dynamics [15, 16], and magnetic solitons [17, 18]. In these studies, the dynamics of magnetization $\mathbf{M}(\mathbf{r}, t)$

is described by a modified Landau-Lifshitz equation including the term of spin-transfer torque. In a typical ferromagnet, the magnetization is rarely uniform, i.e., the spatial-dependence magnetization, and a new form of spin torque [7, 15] is proposed in conducting ferromagnetic structures. With this new form of spin torque, the nonlinear excitations on the background of ground state are studied, such as the unique features of Néel wall motion in a nanowire [15], kink soliton solution and the domain wall dynamics in a biaxial ferromagnet [16], bulk spin excitations [7, 15], and magnetic soliton solutions for isotropic case [17] and uniaxial anisotropic case [18]. It is well known that the nonlinear spin wave and magnetic solitons are always topic research in confined ferromagnetic materials [19, 20, 21, 22] due to the interaction between spin-polarized current and local magnetization, especially the generation and detection of magnons excitation [20] in a magnetic multilayer.

From Eq. (1) one can obtain the solution of spin current if magnetization $\mathbf{M}(\mathbf{r}, t)$ is known. For simplicity we consider an infinite long ferromagnetic nanowire where the electronic current flows along the long length of the wire, defined as x direction. The z axis is taken as the direction of uniaxial anisotropy field and the external field. Assuming the magnetization is nonuniform only along the direction of current, the spin current in Eq. (1) can be written as

$$\mathbf{j}(x, t) = b_J \mathbf{M}(x, t), \quad (2)$$

where $b_J = P j_e \mu_B / (e M_s)$. The spatial variation of the spin current produces a reaction torque on the magnetization as $\tau_b \equiv \partial \mathbf{j} / \partial x = b_J \partial \mathbf{M} / \partial x$ which enter the modified Landau-Lifshitz equation as shown below. As reported in previous work that spin wave solution and soliton solution [7, 15, 18] are admitted for the magnetization $\mathbf{M}(x, t)$, and then the spin current \mathbf{j} has the periodic form and the pulsed form, respectively.

In the present paper, we will investigate the properties of spin current \mathbf{j} on the background of a periodic solution corresponding to the soliton solution of magnetization on a nonlinear spin wave background. The pa-

per is organized as follows. In section II we transform Landau-Lifshitz equation into an equation of nonlinear Schrödinger type in the long-wavelength approximation. By means of Darboux transformation the soliton solution for the spin-polarized current are constructed analytically in section III. In section IV we discuss the properties of the solution for the spin polarized current in detail. Section V is our conclusion.

II. DYNAMICS EQUATION OF MAGNETIZATION IN FERROMAGNETIC NANOWIRE

The dynamics of the localized magnetization is described by the modified Landau-Lifshitz equation [7, 15] with the term of spin-transfer torque as

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\alpha}{M_s} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t} + \boldsymbol{\tau}_b, \quad (3)$$

where the localized magnetization $\mathbf{M} \equiv \mathbf{M}(x, t)$, γ is the gyromagnetic ratio, α is the Gilbert damping parameter, and \mathbf{H}_{eff} represents the effective magnetic field including the external field, the anisotropy field, the demagnetization field and the exchange field. This effective field can be written as $\mathbf{H}_{\text{eff}} = (2A/M_s^2) \partial^2 \mathbf{M} / \partial x^2 + [(H_K/M_s - 4\pi) M_z + H_{\text{ext}}] \mathbf{e}_z$, where A is the exchange constant, H_K is the anisotropy field, H_{ext} is the applied external field, and \mathbf{e}_z is the unit vector along the z direction. Introducing the normalized magnetization as $\mathbf{m} = \mathbf{M}/M_s$, Eq. (3) can be simplified as the dimensionless form

$$\begin{aligned} \frac{\partial \mathbf{m}}{\partial t} &= -(\mathbf{m} \times \frac{\partial^2 \mathbf{m}}{\partial x^2}) + \alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} + \frac{b_J t_0}{l_0} \frac{\partial \mathbf{m}}{\partial x} \\ &\quad - (m_z + \frac{H_{\text{ext}}}{H_K - 4\pi M_s}) (\mathbf{m} \times \mathbf{e}_z), \end{aligned} \quad (4)$$

where time t and space coordinate x has been rescaled by the characteristic time $t_0 = 1/(\gamma(H_K - 4\pi M_s))$ and length $l_0 = \sqrt{2A/((H_K - 4\pi M_s) M_s)}$, respectively.

It is obvious that $\mathbf{m} \equiv (m_x, m_y, m_z) = (0, 0, 1)$ forms the ground state of system, and two types of the nonlinear excited state, i.e., spin wave solution and magnetic soliton, are admitted for Eq. (4). When the magnetic field is high enough, the deviation of magnetization from the ground state is small for two types of excited state. In this case we can make a reasonable transformation

$$\psi = m_x + i m_y, \quad m_z = \sqrt{1 - |\psi|^2}. \quad (5)$$

Substituting the above equations into Eq. (4) we obtain

$$\begin{aligned} i \frac{\partial \psi}{\partial t} &= m_z \frac{\partial^2 \psi}{\partial x^2} - \psi \frac{\partial^2 m_z}{\partial x^2} - \alpha \left(m_z \frac{\partial \psi}{\partial t} - \psi \frac{\partial m_z}{\partial t} \right) \\ &\quad + i \frac{b_J t_0}{l_0} \frac{\partial \psi}{\partial x} - \left(m_z + \frac{H_{\text{ext}}}{H_K - 4\pi M_s} \right) \psi. \end{aligned} \quad (6)$$

It is easy to get two solutions of Eq. (6): one is $\psi = 0$, corresponding to the ground state $\mathbf{m} = (0, 0, 1)$, i.e., $\mathbf{j} = (0, 0, b_J M_s)$, and the other is bulk spin wave excitations, $\psi = A_c e^{i(-k_c x + \omega_c t)}$, corresponding to the periodic spin current

$$\begin{aligned} j_x &= b_J M_s A_c \cos(-k_c x + \omega_c t), \\ j_y &= b_J M_s A_c \sin(-k_c x + \omega_c t), \\ j_z &= b_J M_s \sqrt{1 - A_c^2}, \end{aligned} \quad (7)$$

where k_c and ω_c are the dimensionless wave number and frequency of spin wave, and the transverse amplitude $A_c \ll 1$. For the attractive interaction the nonlinear spin waves in ferromagnet with anisotropy lead to the macroscopic phenomena, i.e., the appearance of spatially localized magnetic excited state (magnetic soliton).

In the present paper we want to obtain the soliton solution of magnetization on a nonlinear spin wave background in a uniaxial ferromagnetic nanowire with spin torque. However, Eq. (6) is not integrable. To our purpose we consider the case of without damping and the long-wavelength approximation [23], where the dimensionless wave number $k_c \ll 1$. Keeping only the nonlinear terms of the order of the magnitude of $|\psi|^2 \psi$, Eq. (6) can be simplified as an integrable equation

$$i \frac{\partial \psi}{\partial t} = \frac{\partial^2 \psi}{\partial x^2} + \frac{1}{2} |\psi|^2 \psi - (1 + \frac{H_{\text{ext}}}{H_K - 4\pi M_s}) \psi + i \frac{b_J t_0}{l_0} \frac{\partial \psi}{\partial x}, \quad (8)$$

whose soliton solutions on the background of the ground state, $\psi = 0$, can be obtained by Hirota methods [18, 24]. In order to discuss the properties of soliton solution on the background of spin wave, here we use a straightforward Darboux transformation [25, 26, 27] to construct general expressions of soliton solution of Eq. (8) with which the soliton solution for the spin-polarized current are obtained from Eqs. (2) and (5). For this reason we will consider mainly the solutions of Eq. (8) in the following section.

The main idea of Darboux transformation is that it firstly transforms the nonlinear equation into the Lax representation, and then in terms of a series of transformations the soliton solution can be constructed algebraically with an obvious seed solution of the nonlinear equation. In terms of Ablowitz-Kaup-Newell-Segur technique Lax representation for Eq. (8) can be constructed as

$$\begin{aligned} \frac{\partial \Psi}{\partial x} &= U \Psi, \\ \frac{\partial \Psi}{\partial t} &= V \Psi, \end{aligned} \quad (9)$$

where $\Psi = (\Psi_1 \ \Psi_2)^T$, the superscript “ T ” denotes the matrix transpose, and the Lax pairs U and V are defined by

$$\begin{aligned} U &= \lambda \sigma_3 + q, \\ V &= (-i2\lambda^2 + \lambda \alpha_1 + \alpha_2) \sigma_3 \\ &\quad + (\alpha_1 - i2\lambda) q + i \left(\frac{\partial q}{\partial x} + q^2 \right) \sigma_3, \end{aligned} \quad (10)$$

where

$$\begin{aligned}\alpha_1 &= \frac{b_J t_0}{l_0}, \alpha_2 = i \frac{1}{2} \left(1 + \frac{H_{\text{ext}}}{H_K - 4\pi M_s} \right), \\ \sigma_3 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, q = \frac{1}{2} \begin{pmatrix} 0 & \psi \\ -\bar{\psi} & 0 \end{pmatrix},\end{aligned}$$

where λ is the complex spectral parameter, and the overbar denotes the complex conjugate. With the natural condition of Eq. (9) $\partial^2 \Psi / (\partial x \partial t) = \partial^2 \Psi / (\partial t \partial x)$, i.e., $\partial U / \partial t - \partial V / \partial x + [U, V] = 0$ the integrable Eq. (8) can be recovered successfully. Now the Eqs. (9) and (10) have made the normal form of the developed Darboux transformation with which we can get the general N -soliton solution as shown in next section.

III. DARBOUX TRANSFORMATION

In this section we briefly introduce the procedure for getting soliton solution for the developed Darboux transformation. Consider the following transformation

$$\Psi [1] = (\lambda I - K) \Psi, \quad (11)$$

where $K = S \Lambda S^{-1}$, $\Lambda = \text{diag}(\lambda_1, \lambda_2)$, and S is a nonsingular matrix which satisfies

$$\frac{\partial}{\partial x} S = \sigma_3 S \Lambda + q S. \quad (12)$$

Letting $\Psi [1]$ satisfy the Lax equation

$$\frac{\partial}{\partial x} \Psi [1] = U_1 \Psi [1], \quad (13)$$

where

$$\begin{aligned}U_1 &= \lambda \sigma_3 + q_1, \\ q_1 &= \frac{1}{2} \begin{pmatrix} 0 & \psi_1 \\ -\bar{\psi}_1 & 0 \end{pmatrix}.\end{aligned}$$

With the help of Eqs. (10), (11) and (12), we obtain the Darboux transformation from Eq. (13) in the form

$$\psi_1 = \psi + 4 K_{12}, \quad (14)$$

which shows that a new solution ψ_1 of Eq. (8) with “seed” solution ψ can be obtained if K is known.

It is verified easily that, if $\Psi = (\Psi_1 \ \Psi_2)^T$ is the eigenfunction of Eq. (9) with eigenvalue $\lambda = \lambda_1$, then $(-\bar{\Psi}_2 \ \bar{\Psi}_1)^T$ is also the eigenfunction, however, with eigenvalue $-\bar{\lambda}_1$. Therefore S and Λ can be taken the form

$$S = \begin{pmatrix} \Psi_1 & -\bar{\Psi}_2 \\ \Psi_2 & \bar{\Psi}_1 \end{pmatrix}, \Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & -\bar{\lambda}_1 \end{pmatrix}, \quad (15)$$

which ensures that Eq. (12) is held. Then Eq. (14) becomes

$$\psi_1 = \psi + 4 (\lambda_1 + \bar{\lambda}_1) \frac{\Psi_1 \bar{\Psi}_2}{\Psi^T \bar{\Psi}}, \quad (16)$$

where $\Psi^T \bar{\Psi} = |\Psi_1|^2 + |\Psi_2|^2$, $\Psi = (\Psi_1, \Psi_2)^T$ to be determined is the eigenfunction of Eq. (9) corresponding to the eigenvalue λ_1 for the solution ψ of Eq. (8). Thus by solving Eq. (9) we can generate a new solution for \mathbf{j} with the help of Eqs. (2), (5) and (8) from an obvious “seed” solution of Eq. (8).

To obtain exact N -order solution, we firstly rewrite the Darboux transformation in Eq. (16) as in the form

$$\psi_1 = \psi + 4 (\lambda_1 + \bar{\lambda}_1) \frac{\Psi_1 [1, \lambda_1] \bar{\Psi}_2 [1, \lambda_1]}{\Psi [1, \lambda_1]^T \bar{\Psi} [1, \lambda_1]}, \quad (17)$$

where $\Psi [1, \lambda] = (\Psi_1 [1, \lambda], \Psi_2 [1, \lambda])^T$ denotes the eigenfunction of Eq. (9) corresponding to eigenvalue λ . Then repeating above the procedure N times, we can obtain the exact N -order solution

$$\psi_N = \psi + 4 \sum_{n=1}^N (\lambda_n + \bar{\lambda}_n) \frac{\Psi_1 [n, \lambda_n] \bar{\Psi}_2 [n, \lambda_n]}{\Psi [n, \lambda_n]^T \bar{\Psi} [n, \lambda_n]}, \quad (18)$$

where

$$\begin{aligned}\Psi [n, \lambda] &= (\lambda - K [n-1]) \cdots (\lambda - K [1]) \Psi [1, \lambda], \\ K_{l_1 l_2} [n'] &= (\lambda_{n'} + \bar{\lambda}_{n'}) \frac{\Psi_{l_1} [n', \lambda_{n'}] \bar{\Psi}_{l_2} [n', \lambda_{n'}]}{\Psi [n', \lambda_{n'}]^T \bar{\Psi} [n', \lambda_{n'}]} - \bar{\lambda}_{n'} \delta_{l_1 l_2},\end{aligned}$$

here $\Psi [n', \lambda]$ is the eigenfunction corresponding to $\lambda_{n'}$ for $\Psi_{n'-1}$ with $\Psi_0 \equiv \Psi$, and $l_1, l_2 = 1, 2, n' = 1, 2, \dots, n-1, n = 2, 3, \dots, N$. Thus if choosing a “seed” as the basic initial solution, by solving linear characteristic equation system (9), one can construct a set of new solutions from Eq. (18).

In the following we take the initial “seed” solution $\psi = A_c e^{i(-k_c x + \omega_c t)}$ corresponding to a periodic solution (7), where the dispersion relation, $\omega_c = k_c^2 - k_c b_J t_0 / l_0 - A_c^2 / 2 + [1 + H_{\text{ext}} / (H_K - 4\pi M_s)]$, is obtained from Eq. (8). After the tedious calculation for solving the linear equation system (9) we have the eigenfunction corresponding to eigenvalue λ in the form

$$\begin{aligned}\Psi_1 &= L C_1 e^{\Theta_1} + \frac{1}{2} A_c C_2 e^{\Theta_2}, \\ \Psi_2 &= \frac{1}{2} A_c C_1 e^{-\Theta_2} + L C_2 e^{-\Theta_1},\end{aligned} \quad (19)$$

where the parameters C_1 and C_2 are the arbitrary complex constants, and the other parameters are defined by

$$\begin{aligned}\Theta_1 &= -\frac{1}{2} i (k_c x - \omega_c t) + D (x + \delta t), \\ \Theta_2 &= -\frac{1}{2} i (k_c x - \omega_c t) - D (x + \delta t), \\ L &= -\frac{1}{2} i k_c - D - \lambda, \\ D &= \frac{1}{2} \sqrt{(i k_c + 2\lambda)^2 - A_c^2}, \\ \delta &= -i 2\lambda - k_c + \frac{b_J t_0}{l_0}.\end{aligned} \quad (20)$$

With the help of the formulas (2), (5), (18), and (19) we can obtain the desired soliton solution for the spin current.

IV. PROPERTIES OF SOLITON SOLUTION FOR THE SPIN-POLARIZED CURRENT

Taking the spectral parameter $\lambda = \lambda_1 \equiv \mu_1/2 + i\nu_1/2$, here μ_1 and ν_1 are real number, in Eq. (19), and substituting them into Eqs. (17) and (5) we obtain the one-soliton solution for the spin-polarized current from Eq. (2) as

$$\begin{aligned} j_x &= b_J M_s [A_c \cos \varphi + \frac{2\mu_1}{\Delta_1} (Q_1 \cos \varphi - Q_2 \sin \varphi)], \\ j_y &= b_J M_s [A_c \sin \varphi + \frac{2\mu_1}{\Delta_1} (Q_1 \sin \varphi + Q_2 \cos \varphi)], \\ j_z &= b_J M_s \sqrt{1 - (A_c + \frac{2\mu_1 Q_1}{\Delta_1})^2 - (\frac{2\mu_1 Q_2}{\Delta_1})^2}, \end{aligned} \quad (21)$$

where

$$\begin{aligned} \theta_1 &= 2D_{1R}x + 2(D_1\delta_1)_R t + 2x_0, \\ \Phi_1 &= 2D_{1I}x + 2(D_1\delta_1)_I t - 2\varphi_0, \\ \varphi &= -k_c x + \omega_c t, \end{aligned} \quad (22)$$

$$\begin{aligned} Q_1 &= A_c L_{1R} \cosh \theta_1 + \left(|L_1|^2 + \frac{1}{4} A_c^2 \right) \cos \Phi_1, \\ Q_2 &= A_c L_{1I} \sinh \theta_1 + \left(|L_1|^2 - \frac{1}{4} A_c^2 \right) \sin \Phi_1, \\ \Delta_1 &= \left(|L_1|^2 + \frac{1}{4} A_c^2 \right) \cosh \theta_1 + A_c L_{1R} \cos \Phi_1, \end{aligned}$$

where the subscript R and I represent the real part and imaginary part, respectively. The other parameters are $D_1 = \sqrt{(ik_c/2 + \lambda_1)^2 - A_c^2/4}$, $L_1 = -ik_c/2 - D_1 - \lambda_1$, $\delta_1 = -i2\lambda_1 - k_c + b_J t_0/l_0$, $x_0 = -(\ln |C_2/C_1|)/2$, and $\varphi_0 = [\arg(C_2/C_1)]/2$, where C_1, C_2 are the arbitrary complex constants.

The solution (21) describes a one-soliton solution for the spin-polarized current in ferromagnetic nanowire embedded in the periodic spin current background (7): (a) When $\mu_1 = 0$, the solution (21) reduces to the periodic solution (7). (b) When the spin-wave amplitude vanishes, namely $A_c = 0$, the solution (21) reduces to the solution in the form

$$\begin{aligned} j_x &= \frac{2\mu_1 b_J M_s}{\cosh \theta_1} \cos(\Phi_1 + \eta), \\ j_y &= \frac{2\mu_1 b_J M_s}{\cosh \theta_1} \sin(\Phi_1 + \eta), \\ j_z &= b_J M_s \sqrt{1 - \frac{4\mu_1^2}{\cosh^2 \theta_1}}, \end{aligned} \quad (23)$$

where

$$\begin{aligned} \theta_1 &= \mu_1 [x + (2\nu_1 + \frac{b_J t_0}{l_0})t + \frac{2}{\mu_1} x_0], \\ \Phi_1 &= \nu_1 \{x - [\frac{1}{\nu_1} (\mu_1^2 - \nu_1^2) - \frac{b_J t_0}{l_0}]t - \frac{2}{\nu_1} \varphi_0\}, \\ \eta &= (1 + \frac{H_{ext}}{H_K - 4\pi M_s})t. \end{aligned} \quad (24)$$

The solution (23) is in fact the same as the solution (8) in Ref. [18] (One should notice that the transformation (5) is different from that in Ref. [18]).

The solution (23) indicates the spatially localized excitation [28] which is denoted by the transverse amplitude $2\mu_1$ deviated from the ground state $\mathbf{j} = (0, 0, b_J M_s)$. The components j_x and j_y precess around the component j_z with the frequency $\Omega_1 = \nu_1^2 - \mu_1^2 + b_J t_0/l_0 + 1 + H_{ext}/(H_K - 4\pi M_s)$, and the soliton solution is characterized by the width $1/\mu_1$, the velocity of soliton center $v_1 = -(2\nu_1 + b_J t_0/l_0)$. The wave number $k_{s,1} = -\nu_1$ and the frequency Ω_1 of the “carrier wave” are related by the dispersion law $\Omega_1 = k_{s,1}^2 - k_{s,1} b_J t_0/l_0 - \mu_1^2 + 1 + H_{ext}/(H_K - 4\pi M_s)$ which shows that the magnetic field contribute to precession frequency only. The magnetic soliton energy is seen to be $E_1 = -b_J^2 t_0^2 / (2l_0)^2 - \mu_1^2 + 1 + H_{ext}/(H_K - 4\pi M_s) + \frac{1}{2} m^* v_1^2$, where the dimensionless effective mass m^* of soliton is $1/2$.

From Eqs. (23) and (24) we also see that the term b_J can change the velocity and the precessional frequency of soliton on a background of the ground state $\mathbf{j} = (0, 0, b_J M_s)$. This case confirms the previous study [15, 18]. However, on the background (7) novel properties of Eq. (21) will be described below. The properties of envelope soliton solution (21) are characterized by the width $1/(2D_{1R})$, the wave number $k_s = -2D_{1I}$, the initial center position $-x_0/D_{1R}$, and the envelope velocity $v_1 = -(D_1\delta_1)_R/D_{1R}$. The initial center position of soliton is moved $x_0(2/\mu_1 - 1/D_{1R})$ by the spin wave which show new way controlling the soliton in space.

From the expressions of D_1 and δ_1 we find that the velocity and the width of envelope soliton are modulated by the amplitude A_c and wave number k_c of spin wave as shown in Figure 1. From figure 1(a) and 1(c) we see that absolute value of velocity and the width of envelope soliton become large with the increasing A_c . Figure 1(b) shows that the value of k_c nearby $-\nu_1$ has obvious effect on the velocity of soliton. When $k_c = -\nu_1$, the width of soliton is maximal as shown in Figure 1(d).

From Eq. (22) we can directly see that when $D_{1I}\delta_{1I} = \delta_{1R}D_{1R}$, the parameters θ_1 depends only on x which implies the envelope velocity $-(D_1\delta_1)_R/D_{1R}$ becomes zero, i.e., the soliton is trapped in space by the nonlinear spin wave. It should be noted that this condition can be written as

$$j_e = \frac{e M_s}{P \mu_B} \frac{l_0}{t_0} \left(-\mu_1 \frac{D_{1I}}{D_{1R}} - \nu_1 + k_c \right), \quad (25)$$

which is determined by the characteristic velocity l_0/t_0 , the amplitude and wave number of soliton and the non-

linear spin-wave, and the parameters $(eM_s/P\mu_B)$, respectively. From a tedious calculation we found that when $A_c \ll k_c$ and $\mu_1 \ll \nu_1$ the condition in Eq. (25) reduces to $j_e \approx -2\nu_1(l_0/t_0)(eM_s/P\mu_B)$. When the amplitude of spin wave vanishes, namely $A_c = 0$, the trapping condition in Eq. (25) reduces to $j_e = -2\nu_1(l_0/t_0)(eM_s/P\mu_B)$ which is determined by the characteristic velocity l_0/t_0 , the soliton wave number ν_1 , and the parameters $(eM_s/P\mu_B)$. These results show that the background has almost no effect on the trapping condition in the special case $A_c \ll k_c$ and $\mu_1 \ll \nu_1$. For the materials of Co₃Pt alloy films [21] which has high perpendicular anisotropy, we chose $H_K = 1 \times 10^4$ Oe, $A = 1.0 \times 10^{-6}$ erg/cm, $4\pi M_s = 1 \times 10^2$ Oe, $\gamma = 1.76 \times 10^7$ Oe⁻¹s⁻¹, $P = 0.35$, and the dimensionless parameters $k_c = 0.05$, $A_c = 0.02$, $\nu_1 = -0.12$, and $\mu_1 = 0.1$. The critical electric current trapping soliton is $j_e = 1.867 \times 10^4$ A/cm². It is very important to point out that from Eqs. (21), (22), and expressions of δ_1 the term b_J change not only the velocity, but also the frequency effecting on soliton energy. This property trapping the soliton in space by the nonlinear spin wave is characterized by the spatial and temporal period along the direction of soliton propagation, $x = -(D_1\delta_1)_R t/D_{1R} - x_0/D_{1R}$, denoted by $\pi(D_1\delta_1)_R / [\delta_{1I}(D_{1R}^2 + D_{1I}^2)]$ and $\pi D_{1R} / [\delta_{1I}(D_{1R}^2 + D_{1I}^2)]$, respectively.

In order to explain some novel properties of solution (21) we discuss the special case $k_c = -\nu_1$ and analyze two representative situations in detail: (a) The amplitude A_c exceeds the half of the transverse amplitude $2\mu_1$ of soliton. (b) The amplitude A_c is less than the half of transverse amplitude $2\mu_1$ of soliton (which implies $A_c, \mu_1 > 0$).

(a) In the case $\mu_1^2 < A_c^2$ the solution (21) reduces to

$$\begin{aligned} j_x &= b_J M_s (R'_1 \cos \varphi - R'_2 \sin \varphi), \\ j_y &= b_J M_s (R'_1 \sin \varphi + R'_2 \cos \varphi), \\ j_z &= b_J M_s \sqrt{1 - A_c^2 - \frac{\zeta_1 (A_c \cosh \theta_1 \cos \Phi_1 - \mu_1)}{(A_c \cosh \theta_1 - \mu_1 \cos \Phi_1)^2}}, \end{aligned} \quad (26)$$

where φ is given in Eq. (22), and the other parameters are determined by

$$\begin{aligned} \zeta_1 &= 4\mu_1 \kappa_1^2, \\ \kappa_1 &= \sqrt{A_c^2 - \mu_1^2}, \\ R'_1 &= -A_c + \frac{2\kappa_1^2 \cosh \theta_1}{A_c \cosh \theta_1 - \mu_1 \cos \Phi_1}, \\ R'_2 &= -\frac{2\mu_1 \kappa_1 \sinh \theta_1}{A_c \cosh \theta_1 - \mu_1 \cos \Phi_1}, \\ \theta_1 &= \mu_1 \kappa_1 t + 2x_0, \\ \Phi_1 &= \kappa_1 (x - v_1 t - 2\varphi_0/\kappa_1). \end{aligned} \quad (27)$$

A simple analysis for Eq. (27) reveals that the solution (26) is periodic in the space coordinate, denoted by $2\pi/\kappa_1$, and aperiodic in the temporal variable, as

shown in Fig. 2. From Fig. 2 we can see that the background becomes unstable, therefore the solution (26) can be considered as describing the modulation instability process [29]. Along the propagation direction of soliton the expression of j_z has a maximum $j_z = b_J M_s$, i.e., $m_z = 1$, at $\cos \Phi_1 = (2\mu_1^2 - A_c^2)/\mu_1 A_c$ when $A_c^2/4 < \mu_1^2 < A_c^2$, which shows these points are not excited even on the spin wave background, and has a minimum $j_z = b_J M_s [1 - (2\mu_1 + A_c)^2]^{1/2}$ at $\sin \Phi_1 = 0$. When $\mu_1^2 < A_c^2/4$, the expression of j_z has a maximum $j_z = b_J M_s [1 - (2\mu_1 - A_c)^2]^{1/2}$ at $\Phi_1 = 0$, and a minimum $j_z = b_J M_s [1 - (2\mu_1 + A_c)^2]^{1/2}$ at $\Phi_1 = \pi$. These results show that the linear combined transverse amplitude of spin wave and magnetic soliton can be obtained in these special cases.

(b) In the case $\mu_1^2 > A_c^2$ the solution (21) reduces to

$$\begin{aligned} j_x &= b_J M_s (R_1 \cos \varphi - R_2 \sin \varphi), \\ j_y &= b_J M_s (R_1 \sin \varphi + R_2 \cos \varphi), \\ j_z &= b_J M_s \sqrt{1 - A_c^2 - \frac{\zeta_2 (\mu_1 - A_c \cosh \theta_1 \cos \Phi_1)}{(\mu_1 \cosh \theta_1 - A_c \cos \Phi_1)^2}}, \end{aligned} \quad (28)$$

where

$$\begin{aligned} \zeta_2 &= 4\mu_1 \kappa_2^2, \\ \kappa_2 &= \sqrt{\mu_1^2 - A_c^2}, \end{aligned} \quad (29)$$

$$\begin{aligned} R_1 &= -A_c + \frac{2\kappa_2^2 \cos \Phi_1}{\mu_1 \cosh \theta_1 - A_c \cos \Phi_1}, \\ R_2 &= \frac{2\mu_1 \kappa_2 \sin \Phi_1}{\mu_1 \cosh \theta_1 - A_c \cos \Phi_1}, \\ \theta_1 &= \kappa_2 (x - v_1 t + 2x_0/\kappa_2), \\ \Phi_1 &= -\mu_1 \kappa_2 t - 2\varphi_0. \end{aligned} \quad (30)$$

With the expressions (28) and (30) we can see the main characteristic of soliton solution: (1) The soliton has the same envelope velocity $v_1 = -(2\nu_1 + b_J t_0/l_0)$ on both the background of a periodic spin current in Eq. (7) and the ground state background $\mathbf{j} = (0, 0, b_J M_s)$. (2) The amplitude of j_z in Eq. (28) has the temporal periodic oscillation as shown in Figure 3. A detail calculation shows that the amplitude of j_z in Eq. (28) has a minimum at $\theta_1 = 0$, which is given by

$$j_z = b_J M_s \sqrt{1 - A_c^2 - \frac{4\mu_1 (\mu_1^2 - A_c^2)}{\mu_1 - A_c \cos \Phi_1}}, \quad (31)$$

and has a maximum

$$j_z = b_J M_s \sqrt{1 - \frac{\mu_1^2 A_c^2 \sin^2 \Phi_1}{\mu_1^2 - A_c^2 \cos^2 \Phi_1}}, \quad (32)$$

at $\cosh \theta_1 = 2\mu_1 / (A_c \cos \Phi_1) - (A_c \cos \Phi_1) / \mu_1$. Fig. 4(a) presents the evolution along the propagation direction of the minimum and maximum intensities given by Eqs. (31) and (32) (see, respectively, the dashed and dotted

lines), and the spin wave intensity (solid line). The location of minimum and maximum amplitude (solid and dotted lines, respectively) in the time-space plane is shown in Fig. 4(b). From Fig. 4, it is seen that the narrower the soliton, the sharper the peak and the deeper the two dips at the wings of the soliton. This feature illustrates the characteristic breather behavior of the soliton as it propagates on the background of a periodic solution for the spin current in ferromagnetic nanowire.

V. CONCLUSION

In summary, by transforming the modified Landau-Lifshitz equation into an equation of nonlinear Schrödinger type, we study the interaction of a periodic solution and one-soliton solution for the spin-polarized current in a uniaxial ferromagnetic nanowire. Our results show that the amplitude of soliton solution has the spatial and temporal period on the background of a periodic

spin current. The effective mass of soliton is obtained. Moreover, we found that the soliton can be trapped only in space. We also analyze the modulation instability and dark soliton on the background of a periodic spin current which shows the characteristic breather behavior of the soliton as it propagates along the ferromagnetic nanowire.

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Figure Captions

Fig. 1. (Color online) Velocity and width variety of soliton solution for spin-polarized current vs the amplitude A_c and wave number k_c of spin wave, respec-

tively. The parameters are $\mu_1 = 0.1$, $l_0 = 2 \times 10^{-8}$ cm, $t_0 = 5.7392 \times 10^{-12}$ s, and $b_J = 52$ cm/s. (1a) Velocity vs the amplitude A_c , $k_c = 0.1$, $\nu_1 = -0.12$ (red line); $k_c = -0.1$, $\nu_1 = 0.12$ (blue dotted line). (1b) Velocity vs spin wave number k_c , $A_c = 0.06$, and $\nu_1 = -0.15$. (1c) Width vs the amplitude A_c , $k_c = -0.1$, $\nu_1 = 0.12$. (1d) Width vs spin wave number k_c , $A_c = 0.06$, and $\nu_1 = -0.15$.

Fig. 2. The evolution of solution (26) with condition $k_c = -\nu_1$, $\mu_1^2 < A_c^2$, and the parameters are $\mu_1 = 0.12$, $\nu_1 = -0.1$, $A_c = 0.16$, $b_J = 34.8$ cm/s, $l_0 = 2 \times 10^{-10}$ m, $t_0 = 5.7392 \times 10^{-12}$ s, $x_0 = -1.27$, and $\varphi_0 = 0$. The spin current j_z is in unit of $b_J M_s$, and

the same as in Fig. 3 and Fig. 4.

Fig. 3. The evolution of solution (28) with the condition $k_c = -\nu_1$ and $\mu_1^2 > A_c^2$. The parameters are $\mu_1 = 0.1$, $\nu_1 = -0.08$, $A_c = 0.06$, $b_J = 41.8$ cm/s, $l_0 = 2 \times 10^{-10}$ m, $t_0 = 5.7392 \times 10^{-12}$ s, $x_0 = 3$, and $\varphi_0 = 0$.

Fig. 4. (a) The evolution of the minimum amplitude of j_z in Eq. (31) (dashed line), the maximum amplitude of j_z in Eq. (32) (dotted line), and the background amplitude of j_z (solid line) in Eq. (7); (b) The location of the minimum amplitude (solid line) and the maximum amplitude (dotted line) in the time-propagation distance plane. The parameters are the same in Fig. 3.

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